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A note on a generalization of a five circle problem

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Abstract. We generalize a problem in Wasan geometry involving three smaller congruent circles touching two larger congruent circles and their common tangent.

Keywords. congruent circles in line

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1. Introduction

In this note we generalize the following problem in [2] (see Figure 1).

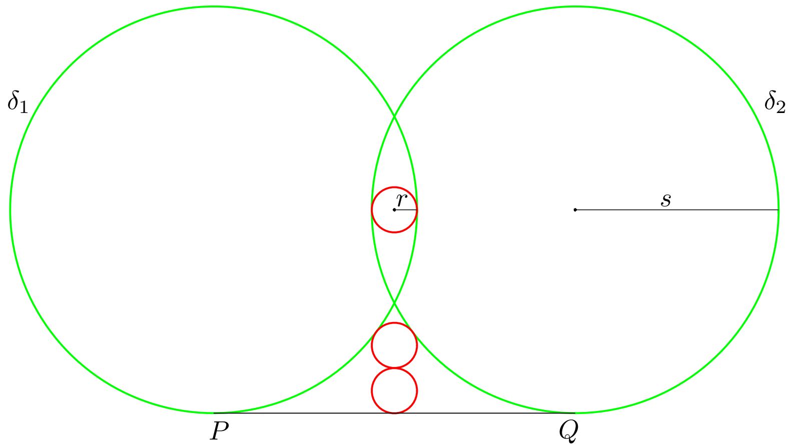

Figure 1: $s = 9r$.

Problem 1. Two intersecting circles δ_1 and δ_2 of radius s touch a segment PQ at P and Q. The maximal circle touching δ_1 and δ_2 from their inside has radius r. A circle of radius r lying inside of the curvilinear triangle made by δ_1, δ_2 and PQ touches δ_1 and δ_2 and the circle touching this circles and PQ at the midpoint also has radius r. Show that $s = 9r$.

A similar problem considered in [1] can also be found in [2].

2. Generalization

We generalize the problem. If γ_1, γ_2, $\cdots$, γ_n are congruent circles such that γ_1 and γ_2 touch, and γ_i touches γ_{i-1} at the farthest point on γ_{i-1} from γ_1 for $i = 3$, 4, $\cdots$, n, then the circles are said to be congruent circles in line. We prove the next theorem (see Figure 2).

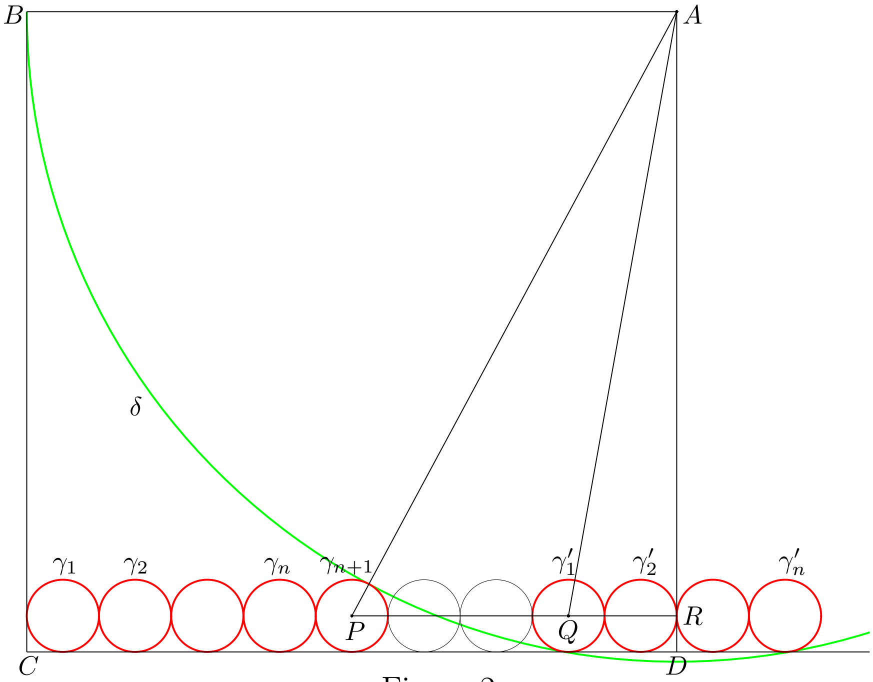

Figure 2.

Theorem 1. *For a rectangle $ABCD$ with $|BC| < |AB| = s$, the circle of radius s and center A is denoted by δ. Let γ_1, γ_2, $\cdots$, γ_{n+1} be congruent circles of radius r in line such that γ_1 touches the segments BC and CD, γ_2 touches CD from the same side as γ_1 and γ_{n+1} touches δ externally. Let γ'_1, γ'_2, $\cdots$, γ'_n be congruent circles of radius r in line such that they touch the line CD from the same side as γ_1 and γ'_1 and γ'_n touch δ internally. The following statements are true.*
(i) $s = 3(n + 2)r$.
(ii) *There are two touching congruent circles of radius r touching CD from the same side as γ_1 such that one touches γ_{n+1}, and the other touches γ'_1.*

Proof. Assume that P and Q are the centers of γ_{n+1} and γ'_1, respectively, and the line PQ meets DA in a point R. From $|AP|^2 - |PR|^2 = |AQ|^2 - |QR|^2$, we have

$$(r + s)^2 - (s - (2n + 1)r)^2 = (s - r)^2 - ((n - 1)r)^2.$$

This implies $s = nr$ or $s = 3(n + 2)r$. Therefore we get (i), since $s > nr$. The part (ii) follows from the fact $s - 2(n + 1)r - nr = 4r$. □

References

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[2] No author name, Enri Shinjutsu (圓理新術), no date, Digital Library Department of Mathematics Kyoto University.