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Solution to 2018-2 Problem 1

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Abstract. A solution to 2018-2 Problem 1 is given.

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1. Problem 1

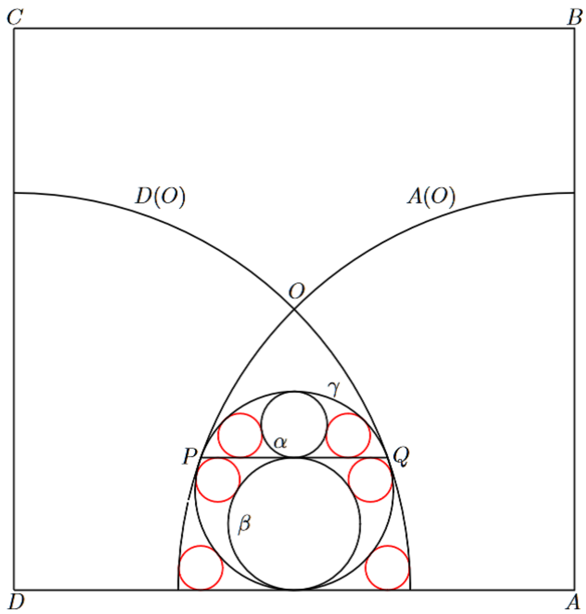

Figure 1. The problem

Let $ABCD$ be a square with center O (see Figure 1). γ is a circle touching the side DA from the inside of $ABCD$ and the circle $A(O)$ and $D(O)$ internally at points

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P and Q, respectively, where $A(O)$ is the circle with center A passing through O and similarly $D(O)$ is defined. Circles touching the segment PQ at the midpoint and γ internally are denoted by α and β. Prove or disprove that all the incircles of the curvilinear triangles made by α, γ and PQ, the curvilinear triangles made by β, γ and PQ, the curvilinear triangle made by $A(O)$, γ and DA are congruent.

2. Solution

We assume that $|PA| = 1$, γ has radius r_3 and center S_3. Then $|EA| = \frac{\sqrt{2}}{2}$, $r_3^2 + \frac{1}{2} = |S_3A|^2$ and $|S_3A| + r_3 = 1$. Hence we get $r_3 = \frac{1}{4}$ and $|S_3A| = \frac{3}{4}$.

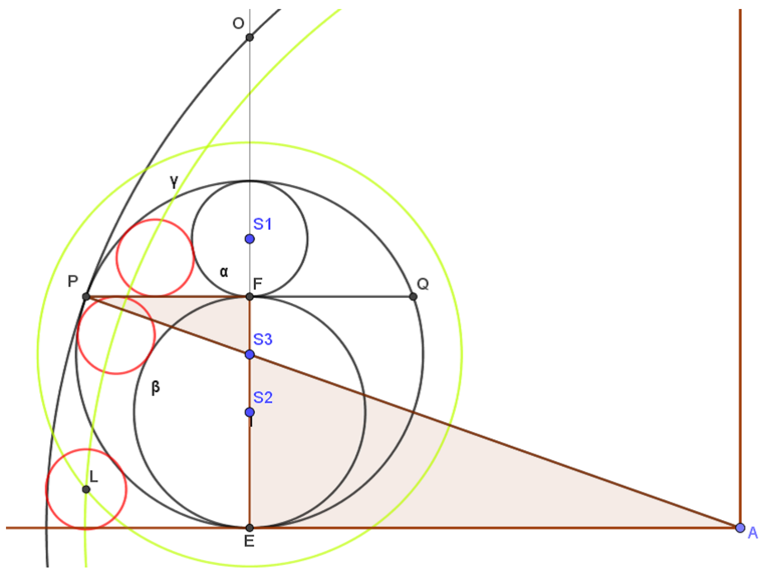

FIGURE 2.

Triangles ΔEAS_3 and ΔFPS_3 are similar (see Figure 2). So $|S_3A|/r_3 = r_3/|S_3F|$. From this we get $|S_3F| = \frac{1}{12}$. Therefore the radius of the circle β is $r_2 = \frac{1}{2}(r_3 + |S_3F|) = \frac{1}{6}$. And the radius of the circle α equals $r_1 = r_3 - r_2 = \frac{1}{12}$.

The first and second incircles of the curvilinear triangles in the problem are well-known Archimedean twin circles, so their radii are equal to $\frac{r_1 r_2}{r_1+r_2} = \frac{1}{18}$.

To consider the radius of the third incircle, we use a Cartesian coordinate system with origin E such that S_3 have coordinates $(0, 1/4)$. Then we can assume that the circle $A(O)$ has center $(\sqrt{2}/2, 0)$.

We may assume that the third incircle has center (x, t) and radius t. Then we have

$$\left(x - \frac{\sqrt{2}}{2}\right)^2 + t^2 = (1-t)^2,$$
$$x^2 + \left(t - \frac{1}{4}\right)^2 = \left(\frac{1}{4} + t\right)^2.$$

Eliminating x and solving the resulting equation, we get $t = \frac{1}{18}$.