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Problems 2017-2

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We propose two problems arising from figures appeared in sangaku problems in Fukushima [1].

Problem 1. Let E and F be points on the side DA of a square $ABCD$ (see Figure 1). Let δ_1 be the incircle of the triangle ABE touching BE and AE at points G and H. Also let δ_2 be the incircle of the triangle DCF touching CF and DF at points I and J. Prove or disprove the followings:

(i) The lines GH and IJ meet in the center of $ABCD$.

(ii) Let ε be the incircle of the triangle made by the lines BC, BE and CF. Then ε touches the remaining external common tangent of δ_1 and δ_2.

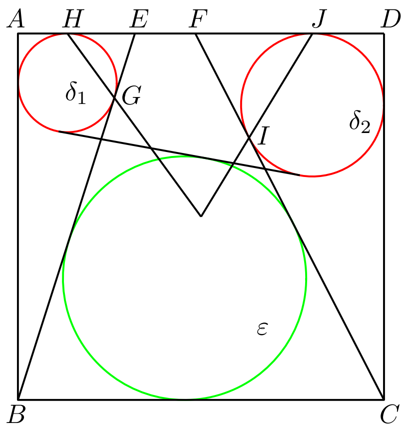

Figure 1.

Remark. Let d_i and e be the radii of δ_i and ε, respectively, and let $s = |AB|$. There is a sangaku problem stating that the relation

$$\frac{1}{e} = \frac{1}{s - 2d_1} + \frac{1}{s - 2d_2}$$

holds [1]. The problem was proposed by Takagi (or Takaki) (高木甚六) in 1877 [1, p. 44].

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Problem 2. Let γ be a circle of radius c with diameter BC for a rectangle $ABCD$ with $|AB| > c$ (see Figure 2). The remaining tangent of γ from A intersects the side CD in a point E. Circles δ_1, δ_2, ε_1, ε_2 and ε_3 are defined as follows:

δ_1: incircle of the curvilinear triangle made by AB, AE and γ, where the common internal tangent of γ and δ_1 intersects DA and AE at points F and G.

δ_2: incircle of the triangle AED touching DE and AE at points P and Q.

ε_1: incircle of the triangle AFG.

ε_2: incircle of the curvilinear triangle made by γ, δ_1 and AB touching AB and γ at points R and S.

ε_3: incircle of the curvilinear triangle made by CE, AE and γ touching CE and γ at points T and U.

We assume that the circles δ_1 and δ_2 are congruent. Prove or disprove the followings:

(i) The circles ε_1, ε_2 and ε_3 are congruent.

(ii) The lines PQ, RS, TU meet in a point H.

(iii) The distances from H to AB, CD and DA are the same and equals c in the occasion (ii) being true.

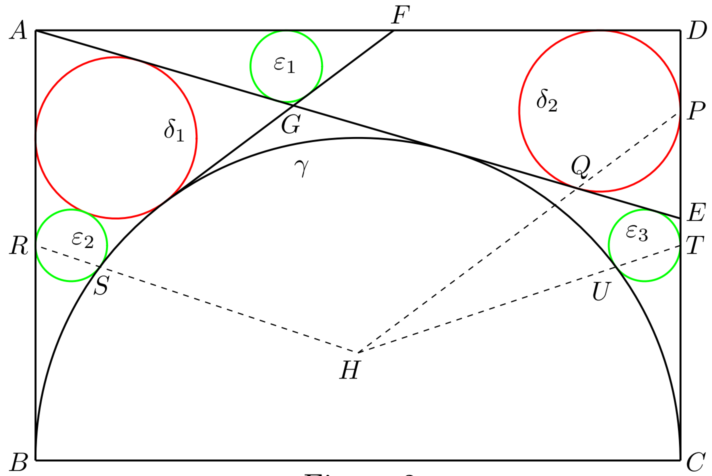

Figure 2.

Remark. Let d be the common radius of δ_1 and δ_2. There are several sangaku problems stating that the radius of ε_3 equals $4d/9$ [1, pp. 263, 310, 315], while $d = c/4$ holds [2, 3, 4]. Notice that (iii) and (i) of Problem 1 assert similar things in a sense.

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