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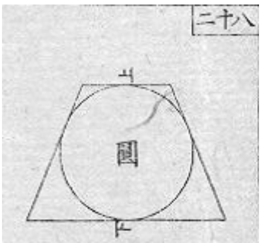
Problems 2020-1

Sampō Jojutsu is a popular Wasan book of geometric formulas presenting formulas arising from one hundred and two geometric figures [2]. Each of the figures from the eighty second to the last is involving an ellipse. We propose the first half of them as problems here.

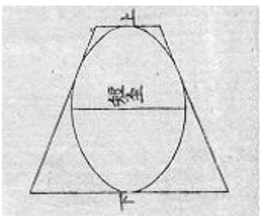
Prove or disprove the following relations in the problems, and send a solution with a generalization or an interesting idea. There is no deadline.

We denote the semi-major axis and semi-minor axis of the ellipse in each of the problems by a and b, respectively

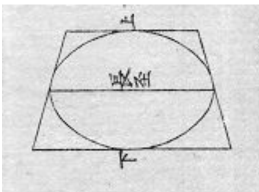
Problem 1. Let s and t be the lengths of the parallel sides of the isosceles trapezoid in the figure.
(1) If r is the inradius of the isosceles trapezoid, then $st = 4r^2$.



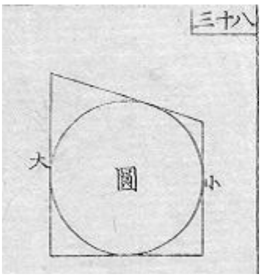
(2) $st = 4b^2$.



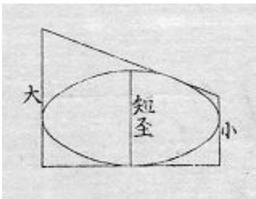
(3) $st = 4a^2$.



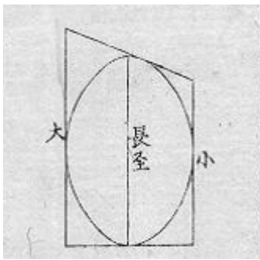
Problem 2. Let s and t be the lengths of the parallel sides of the trapezoid in the figure.
(1) If r is the inradius of the trapezoid, then $r = \dfrac{st}{s+t}$.



(2) $b = \dfrac{st}{s+t}$.

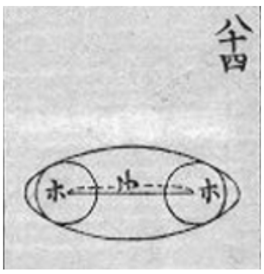


(3) $a = \dfrac{st}{s+t}$.



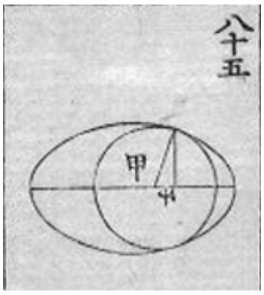
Problem 3. If r is the radius of the two congruent circles, and d is the distance between the centers of the circles in the figure, then

$$d = \frac{2\sqrt{(a^2-b^2)(b^2-r^2)}}{b}.$$



Problem 4. If r is the radius of the circle, and d is the distance between the center of the circle and the foot of perpendicular to the major axis from one of the points of tangency of the ellipse and the circle in the figure, then

$$d = b\sqrt{\frac{b^2 - r^2}{a^2 - b^2}}.$$

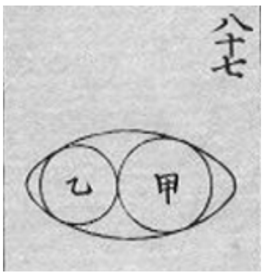


Problem 5. If r_s (resp. r_l) is the radius of the smallest (resp. largest) circle touching the ellipse from outside (resp. inside) of the ellipse internally at one of the points of intersection of the ellipse and the minor (resp. major) axis in the figure, then

$$a^2 = br_s \ \textit{ and } \ b^2 = ar_l.$$

Problem 6. If r_1 and r_2 are the radii of the two circles in the figure, then

$$\sqrt{a^2 - b^2}\left|\sqrt{b^2 - r_1^2} - \sqrt{b^2 - r_2^2}\right| = b(r_1 + r_2).$$

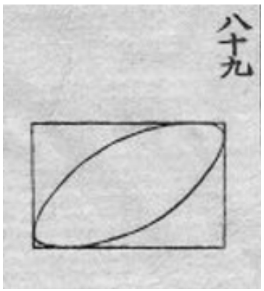


Problem 7. Let s be the side length of the rectangle touching the ellipse, and let t ($t < s$) be the side length of the rectangle which does not touch the ellipse. If the major axis of the ellipse lies along one of the diagonals in the figure, then

$$4(t^2a^2 + s^2b^2) = t^2(s^2 + t^2).$$

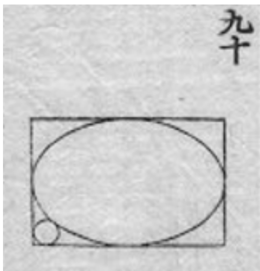
Problem 8. If s and t are the side lengths of the rectangle in the figure, then

$$4(a^2+b^2)=s^2+t^2.$$



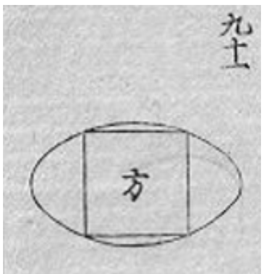
Problem 9. If r is the radius of the small circle in the figure, then

$$r^2-2r(a+b+\sqrt{ab})+ab=0.$$



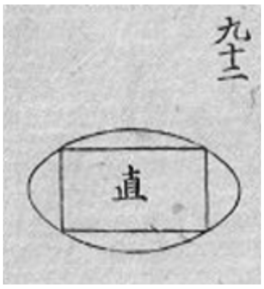
Problem 10. If s is the side length of the square in the figure, then

$$s\sqrt{a^2+b^2}=2ab.$$



Problem 11. If s and t ($t<s$) are the side lengths of the rectangle in the figure, then

$$t^2a^2+s^2b^2=4a^2b^2.$$



All the figures are taken from [1].

REFERENCES

[1] H. Okumura (奥村博) ed., Wasansho Shūsei (和算書集成) (Wasan collection), 2001, Iwanami Shoten.
[2] Yamamoto (山本賀前), Sampō Jojutsu (算法助術), 1841.